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STOCK MODEL WITH VARIABLE PRICES AND KNOWN PROBABILITY

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Abstract: Inventory problem is becoming more relevant in the modern economy. The problem arises as a problem of raw material supplies and finished goods inventory problem. Other group of problems is not treated isolate, but also includes broader aspects associated with the business. Due to the nature of the problem one general model which includes all solutions optimize inventory. General formulation solutions can be expressed in terms of determining the volume of stocks at minimal costs. This request seems simple but to set up and resolve the problem can be complex, and depends on a large number of elements. First of all it is necessary to specify the costs but we must take into account the different production organization. This paper discusses with varying prices and with known probability.

Keywords: *stocks, model, price, probability*.

1. INTRODUCTION

Among other models the stock model is interesting inventory model with variable prices and with a known probability. In addition to the already observed costs which are taken into account explicitly and with other elements, primarily with observed prices of products and probability of certain activities. We will consider the problem of optimizing inventory provided with the price of known probability figures explicitly for this model. Inventory model with a known probability is always possible to apply to the companies' vehicle fleet and, when a company uses a large number of identical vehicles due to the failures vehicles are often out of service. If a company has spare parts in stock, repairs can be carried out without delay. The lack of spare parts causes losses.

2. INVENTORY MODEL WITH VARIABLE COST

We define the elements of the model:

a) c_2 – fixed costs for one order;

b) px– costs - or order value of x units:

c) $c_2\frac{xT}{2k}r$- costs of studying in the interval Δt ;

d)$px\frac{xT}{2k}r$- costs of studying for the px in the interval Δt, gdje je $\frac{xT}{2k}$ correction factor in the interval Δt, i.e.

$$\frac{x}{2}\cdot\frac{\Delta t}{x}=\frac{\Delta t}{2}=\frac{xT}{2k}.$$

Sum of the cost of a) to d) giving the cost in the interval Δt , i.e.

$$c_2+px+c_2\frac{xT}{2k}r+px\frac{xT}{2k}r, \qquad (1)$$

and the total cost for the whole period we get after multiplying (1) the number of orders $\frac{k}{x}$,

respectively

$$C=\frac{c_2k}{x^2}+pk+\frac{c_2T}{2}r+\frac{pxT}{2}r. \qquad (2)$$

The optimum size of the order is determined from x

$$C'=-\frac{c_2k}{x^2}+\frac{pT}{2}r,$$

where, after solving, $C'=0$, we have

$$x_{opt}=\sqrt{\frac{c_2}{p}\cdot\frac{2K}{rT}}. \qquad (3)$$

with minimal costs

$$C_{min}=\sqrt{2rpc_2kT}+pk+\frac{c_2T}{2}r, \qquad (4)$$

Wherein $C''(x_{opt})=\frac{2c_2k}{x^3}>0.$

When unchanged price p, the number of orders in the period T is

$$\frac{k}{x_{opt}}=\sqrt{\frac{r}{2}\cdot\frac{p}{c_2}\cdot kT}, \qquad (5)$$

and the interval between orders

$$\Delta t=\frac{T}{k}x_{opt}=\sqrt{\frac{2}{r}\cdot\frac{c_2}{p}\cdot\frac{T}{k}} \qquad (6)$$

The results are significant primarily because in them the product price figures explicitly. We note that the other conditions of constant size order stands in inverse proportion to the price, that is if the price of the product is higher, the more order is smaller in scale (3), and vice versa. From (5) what we see that the number of orders commensurate with the price, and is higher when the price is higher[1].

We use the results in one special case, when the price p varies depending on the size of the order. In fact, it is often the case that depending on the size of order gives a discount, so that the customer is facing a number of alternative options in terms of making the final decision. Let us examine some of these options[2].

[1] Charnes A., Cooper W. and Henderson A.: An Introduction to Linear Programming, John Wiley, New York, 1956.

[2] Dantzig G.B.: Linear Programming and Extensions, Princenton University Press, Princeton, 1963.

Denote by x = m size of orders for which the manufacturer gives a discount. In other words, producer sells product X at a price p_1 for every order that is in scope if it is less than m units, at a price $p_2 < p_1$ for orders that are equal to or greater than m units[3].

According to expression (3), it follows that the optimal orders volume for different p stand in the following ratio:

$$x_{opt}(p_1) < x_{opt}(p_2) za p_1 > p_2 \qquad (7)$$

ie, generally

$$x_{opt}(p_1) < x_{opt}(p_2) < \cdots < x_{opt}(p_n) \qquad (8)$$

$$za p_1 > p_2 > \cdots > p_n .$$

On the other hand, the total costs expressed by equation (2) for x <mix> m units are as follows:

$$C(x) = \frac{c_2 k}{x} + p_1 k + \frac{c_2 T}{2} r + \frac{p_1 x T}{2} r \qquad (9)$$

$$C(m) = \frac{c_2 k}{m} + p_2 k + \frac{c_2 T}{2} r + \frac{p_2 m T}{2} r. \qquad (10)$$

Comparing (9) and (10) we get the following equality

$$\left(\frac{c_2 k}{x} + p_1 k + \frac{c_2 T}{2} r\right) > \left(\frac{c_2 k}{m} + p_2 k + \frac{c_2 T}{2} r\right), \qquad (11)$$

while for the last members $\frac{p_1 x T}{2} r$ i $\frac{p_2 m T}{2} r$ can not say anything definite.

3. INVENTORY MODEL WITH A KNOWN PROBABILITY

Defineswhat are the general criteria for determining the optimum.

Let's start with a cost function:

$$C(z) = \alpha \sum_{r=0}^{z} (z-r)P(r) + \beta \sum_{r=z+1}^{\infty} (r-z)P(r)$$

and determine

$$C(z+1),$$

i.e.

$$C(z+1) = \alpha \sum_{r=0}^{z+1} (z+1-r)P(r) + \beta \sum_{r=z+2}^{\infty} (r-z-1)P(r)$$

Inventory model with a known probability can be viewed in two special cases. First, when the volume of the stock is $(\alpha + \beta)\, P(r < z_0) - \beta = 0$, resulting $C(Z_0 + 1) = C(z_0)$. According to thse equations we have[4]:

[3] I. Fischer: Mathematical Investigations in the Theory of Value and Prices, 1892.

[4] A. Conrnot : Recherches sur les principes matématiques de la thèorie des richesses, 1838.

$$P(r < z_0) = \frac{\beta}{\alpha + \beta}$$

or

$$P(r \leq z_0 - 1) < \frac{\beta}{\alpha + \beta} = P(r < z_0)$$

Second, when the volume of stock z_0 such as

$$P(r < Z_0 - 1) = \frac{\beta}{\alpha + \beta}$$

$$P(r < Z_0 - 1) = \frac{\beta}{\alpha+\beta} < P(r < z_0).$$

In both cases, the optimum value of z is not unambiguous, but occurs as an alternative$z_0$or $z_0 + 1\, First, and as z_0 - 1orz_0$in the second, with the same minimum cost.

The problem can be seen in the case of a continuous basis, so that costs for continuous aleatory can be written as.[5]

$$C = \alpha \int_0^z (z - r) f(r)\, dr + \beta \int_z^\infty (r - z) f(r)\, dr$$

Where is$\int_{r1}^{r2} f(r)\, dr$ probability of occurrence of events in the interval (r_1, r_2), and $F(z) \int_0^z f(r) dr$.

By this tool it is possible to determine the optimal value z, for which costs are minimal. Note that this is achieved by differentiating the integral of the parameter that is for

$$L(x) - \int_{a(x)}^{b(x)} f(x, y) dy$$

We have:

$$\frac{dL}{dx} - \int_{a(x)}^{b(x)} \frac{\sigma f(x, y)}{\sigma x} dy + f(b, x) \frac{db}{dx} - f(a, x) \frac{da}{dx}$$

For the cost function we have:

$$\frac{dL}{dz} = \infty \int_0^z f(r)\, dr - \beta \int_z^\infty f(r)\, dr$$

$$= \propto F(z) - \beta(1 - F(z))$$

$$= (\alpha + \beta) F(z) - \beta$$

As α and β are not simultaneously equal to zero and $f(z_0) \geq 0$,there will be a requirement for a minimum of C functions to be fulfilled in both cases (greater than or equal to zero), with what for$f(z_0) + 0$function $f(r) > 0$, as continuous function, must have zero as a minimum forz_0.

[5] A. Marshall: Principles of Economics, MacMilon and Co., London 1890.

CONCLUSION

The general formulation of the problem can be expressed as follows: determine the extent of the stock so that the corresponding costs are minimal. While the application is setup relatively simple the troubleshooting can be complex, because it depends on many elements. Specifically, in the first place, it is necessary to specify which costs are the issues, whether referring only stocking costs, or other types of costs in the production process. Production costs are different for different organization of production, production of different series and so on. In other words, the complexity stock problem stems from the fact that simultaneously must be monitored technical - technological relationships with elements of the market (supply and demand) in a given time interval.

MODEL ZALIHA SA PROMJENJIVOM CIJENOM I POZNATOM VJEROVATNOĆOM

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Apstrakt: Problem zaliha postaje sve relevantniji u modernoj ekonomiji. Problem se javlja kao problem zalihe sirovina i problema zaliha gotove robe. I jedna i druga grupa problema ne tretiraju se u izolaciji, negouključuju i šire aspekte povezane s poslovanjem. Zbog prirode problema postoji opšti model koji uključuje sva rješenja za optimizaciju zaliha. Opšti formulacija rješenja može se izraziti u smislu određivanja volumena zaliha uz minimalne troškove. Ovaj zahtjev se čini jednostavan, ali postavljanje i riješavanje problema može biti kompleksno, i ovisi o velikom broju elemenata. Na prvom mjestu je potrebno odrediti troškove, ali moraju se uzeti u obzir različite proizvodne organizacije. U radu se raspravlja o slučaju zaliha sa različitim cijenama i sa poznatom verovatnoćom.

Ključne riječi: *zalihe, model, cijena, vjerovatnoća.*

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