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USE OF THE MONTE CARLO SIMULATION IN VALUATION OF EUROPEAN AND AMERICAN CALL OPTIONS

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Abstract: This study examines the valuation methods used for pricing European and American call options. Options are financial instruments that play an important role in the financial industry and are used in hedging, speculating and arbitraging. Because options are widely used in investing, there is a need for valuation methods that are as precise as possible. Options have been perceived as obscure financial instruments due to the lack of valuation techniques in the past. However, with the discovery of Black-Scholes Model in 1973, the first option valuation method, option trading escalated. In this thesis, the fair market value of S&P 500 index with European exercise style, The Google Option Contract and Apple Option Contract will be obtained by using the Black-Scholes Model, the General Monte Carlo Simulation, The Combined Method and the Least-Squares Monte Carlo. The results from three models will be compared to determine the best valuation method.

Keywords: *options, valuation, Monte Carlo Simulation*

JEL Classification Code: F30, F37 FIN

The main aim of this study is to compare the results obtained from different call option pricing models. The properties of Google Option Contract, SPX Option Contract and Apple Option Contract have been used to facilitate the valuation process. Three different valuation cases are examined and their results are presented. This study hypothesizes that the Monte Carlo Method (general) is the most efficient valuation tool due to its flexibility.

Due to the fact that the main goal of every trader is to make a profit, it is clear that traders want to buy options below their fair value and to sell options when they exceed their fair value. As previously discussed, the valuation of option price is an active area of research in academia and the financial industry, and there are many researchers who are trying to discover the option valuation tool that will price options accurately. Since options are complex financial instruments, it is essential to observe all factors that have an impact on option prices. Those factors are the following: 1) the current stock price (So); 2) the strike price (K); 3) the time to expiration (T); 4) the volatility of the stock price (σ); 5) the risk-free interest rate (r); and 6) the dividends that are expected to be paid. The predicted effect on the value of an option caused by an increase in one of these six variables while keeping the rest fixed is summarized in Table 1.

Table 1: Summary of the effect on the price of a stock option caused by increase in one factor

Variable	*European call*	*European put*	*American call*	*American put*
Current stock price	+	-	+	-
Strike price	-	+	-	+
Time to expiration	**?**	**?**	+	+
Volatility	+	+	+	+
Risk-free rate	+	-	+	-
Amount of future dividends	-	+	-	+

+ indicates that an increase in the variable causes the option price to increase or stay the same;

-indicates that an increase in the variable causes the option price to decrease or stay the same;

? indicates that the relationship is uncertain

Source: (Hull, 2012, p.235)

When a call options is exercised, the payoff is equal to the difference between the current underlying stock price and the strike price. The increase in the current underlying stock price, while all other variables are held constant, will lead to increase in the value of call options. On the other hand, an increase in the strike price, while all other variables are held constant, will bring the option value down because net payoff will decrease.

Volatility can be perceived as the most challenging variable that is involved in option valuation. Hull (2012) writes that “the volatility of a stock price is a measure how uncertain we are about the future stock price movements” (p.235). Investors that held call options profit from price increases. In the case of a price decrease, investors that hold call options experience a limited loss because the most that they can lose is the premium paid to purchase option rights since they won`t exercise the option in this case. There is the Risk-Return Tradeoff between the increase in volatility and the option value. The increase in the volatility, will lead to a potential increase in the value of call options.

Dividends have the opposite effect on the call and put options. In particular, the existence of future dividends will lead to a decrease in the underlying stock price of a call option. The current underlying stock price of a call option will be decreased by the amount of dividend paid. A decrease in the underlying stock price, while all other variables are fixed, will lead to a decrease in the value of call options.

An increase in the Risk-free interest rate, keeping everything else fixed, will increase the call option value. Hull (2012) writes that “when interest rates in the economy increase, the expected return required by investors from the stock tends to increase” (p.237). Furthermore, the author states that an increase in interest rates will decrease the present value of the future cash flows that option holders receive (p.237). Those two effects are combined and they together have a positive impact on the value of call options.

American call options become more valuable with the increase in time to expiration. The longer time horizon means that there is more time for some positive or negative event to occur and have an impact on the option value. When it comes to European call options, the impact of time to expiration is undetermined. Hull (2012) gives a great example that illustrates these phenomena. He assumes the existence of two options: Option A which expires in one month and Option B which expires in two months. Furthermore, Hull assumes that there is an expected large dividend in the next six weeks. The expected dividend will impact option B – there will be a decrease in the underlying stock price and value of Option B. In this case, the one-month Option A will be more valuable than the two-month Option B.

RESEARCH AIM AND HYPOTHESIS

The main goal of this study is to investigate the analytical and mathematical methods used in estimation of the fair market value of the European and American call options. All algorithms are created in Python which is a free and powerful programming language with a lot of packages that are widely used in finance (see Appendix). In fact, Python is commonly used in investment banking, risk management, analyzing data and building different models. There are many libraries such as Numpy, Scipy and pandas that are very useful when dealing with scientific computing. It is interesting to point out that majority of options belong to the American exercise style. Options on stock indices and commodities, however, are mostly European exercise style. In order to perform the empirical work, a Google Option Contract, Apple Option Contract and SPX Option Contract parameters are used. Google and Apple option contracts both belong to the American exercise style. Google option does not pay any dividend in contrast to the Apple option contract. So, I taught it would be interesting to examine two option that are in the same sector (technology) and have different characteristics. SPX Option Contract generally mirrors the performance of 500 leading US firm and it is very popular investment tool for risk averse investors.

Case 1: Empirical Work, Methodology and Results for American Call Option with no Dividends (GOOGLE)

An American call option that does not pay dividends is considered to be the simplest type of American options due to the fact it requires only a simple payoff calculation. Although American call options can be exercised at any time during their life time, it is not optimal to exercise them until the options` maturity because the value of American call option increases with the increase in time horizon (recall Table 1). In fact, the valuation of American call options that do not pay dividends should be identical to the valuation of European call options due to the fixed exercise time recommended. So, an European call option can be valuated using the same pricing models for an American call option without dividends. The following valuation models have been used in to price American call options that do not pay dividends: 1) The Black Scholes Model; 2) The Monte Carlo Method; and 3) A Method that is a combination of the-Black-Scholes Model and the Monte Carlo Method.

The empirical work is based on input of Google Option Contract parameters in algorithms built based on The Black Scholes Model and The Monte Carlo Method (refer to Appendix). The results of simulation prices of an American call with no dividends are presented in Table 2. When an option is in-the-money (stock price > strike price), all three methods produce approximately the same results of option fair market value. In fact, the difference between the results of three methods (BSM = 237.36, MC = 237.42, Combined=237.34) presented below when simulations are iterated 400,000 times does not exceed 1 %. So, it is possible to conclude that the Monte Carlo Method and Combined Method benefit greatly with the increase in iteration. Furthermore, if we observe The Monte Carlo Method and Combined Method when Google Option Contract is at-the-money, it is possible to notice that at the low iteration level (I = 100) that the Combined Method (237.43) performs better in comparison to the General Monte Carlo Method (235.30). In fact, the difference between The Black Scholes Model (237.36) and Combined Model (237.43) is only 0.03%.

Table 2:

Simulation Prices American Call in-the-money with no dividends

S0 = 837.17, K=600, T=0.0198, σ=27.15 %, r= 1.56 %, I = # of iterations		
BSM Analytical	**General Monte Carlo**	**Combined**
237.36	235.30 (I = 100) 237.01 (I = 10,000) 237.33 (I = 100,000) 237.40 (I = 200,000) 237.42 (I = 400,000)	237.43 (I = 100) 237.49 (I = 10,000) 237.41 (I = 100,000) 237.38 (I = 200,000) 237.34 (I = 400,000)

(GOOGLE)

Furthermore, from **Chart 1** it is possible to see that with the increase of the number of iterations, The Monte Carlo Method and The Combined Method converge. The reason for this convergence is the Law of Large numbers- as the number of iterations increases, the actual result will converge to its theoretical value.

Chart 1: In-the-Money Google Option Contract

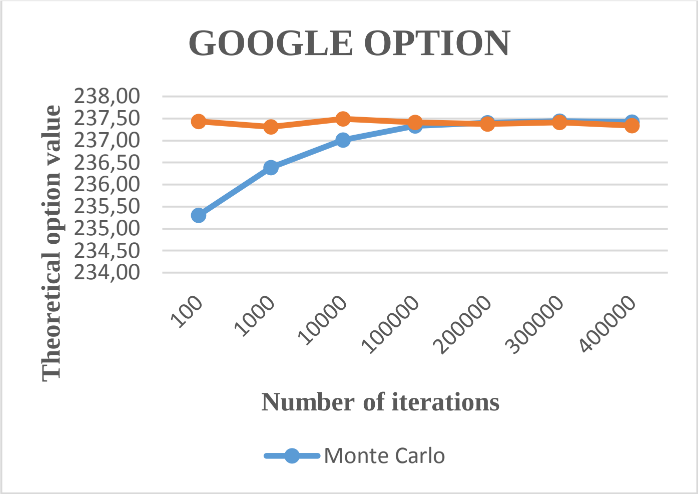


Table 3 is used to make qualitative statements and better visual comparison of fair market values of the Google option contract produced by alternative formulas (General Monte Carlo Method and Combined Method) relative to the values of those same options according to the Black Scholes Model.

Let consider the market value when the Google option contract is at-the-money (Stock price = Strike price). When expiration time is five days, the market value is estimated to be the following: The Black Scholes Model (12.90), The General Monte Carlo (12.92) and Combined Method (12.94). The greatest difference is between The Black Scholes Model and Combined Method – 0.31 %. When expiration time is 252 trading days, at-the-money Google contract`s market value is estimated as following: BSM (96.33) , MC (96.50) and Combined (96.51). Again, the greatest difference between two models is between The Black Scholes Model and Combine Model, but it is insignificant (0.19 %). So, it is possible to conclude that for at-the-money American call options that do not pay dividends – all three methods are equally good predictors of the option value.

When the Google option contract is out-of-money, for the short expiration time (five and ten days) all three models predict that the market value will be equal to 0. With the increase in time to expiration (126 and 252 days), the low market value is estimated and it is approximately the same for all three models. Out-of-money option is going to expire being worthless and the investor will lose his initial premium paid to purchase the call option rights.

Table 3: Comparative Option Values GOOGLE

$S0 = 837.17$, $r=1.56\%$, $\sigma = 4.67\ \%$, Iterations = 400,000

	Time to Expiration (in days)											
Strike Price	5	10	126	252	5	10	126	252	5	10	126	252
	BSM				**Monte Carlo Simulation**				**Combined**			
100	737.20	737.23	737.95	738.72	737.26	737.32	738.25	739.14	737.22	737.26	738.00	738.56
500	337.32	337.48	341.18	346.56	337.39	337.57	341.48	346.95	337.32	337.43	341.08	346.73
600	237.36	237.54	243.89	255.39	237.42	237.63	244.16	255.76	237.40	237.47	243.49	255.62
837.17	12.90	18.32	67.09	96.33	12.92	18.35	67.20	96.50	12.94	18.32	67.23	96.51
1200	0.00	0.00	2.50	13.05	0.00	0.00	2.53	13.11	0.00	0.00	2.45	13.02

Guide: Table 3 provides answers to following two questions:
1.If fixed strike price is given, how will time to expiration impact the market value of call option?
2. If fixed time to expiration is given, how will strike price impact the market value of call option?

Case 2: Empirical Work and Methodology for European Call Options with Dividends (SPX)

SPX is the option contract that has the S&P 500 index as underlying assets. According to CBOE

data, SPX is the most actively-traded option contract in the US. SPX Options have European exercise style which means that contract will be exercised only at the maturity.

Due to the fact that SPX has a constant dividend yield of (0.0204), it is important to adjust the underlying stock price for the present value of the dividend amount. In order to make adjustments, the dividend amount has to be discounted by using dividend discount model. The dividend discount model equation is expressed as PV = $\frac{D}{r-g}$ (D is total dividend amount, r is risk free rate, g is dividend growth).

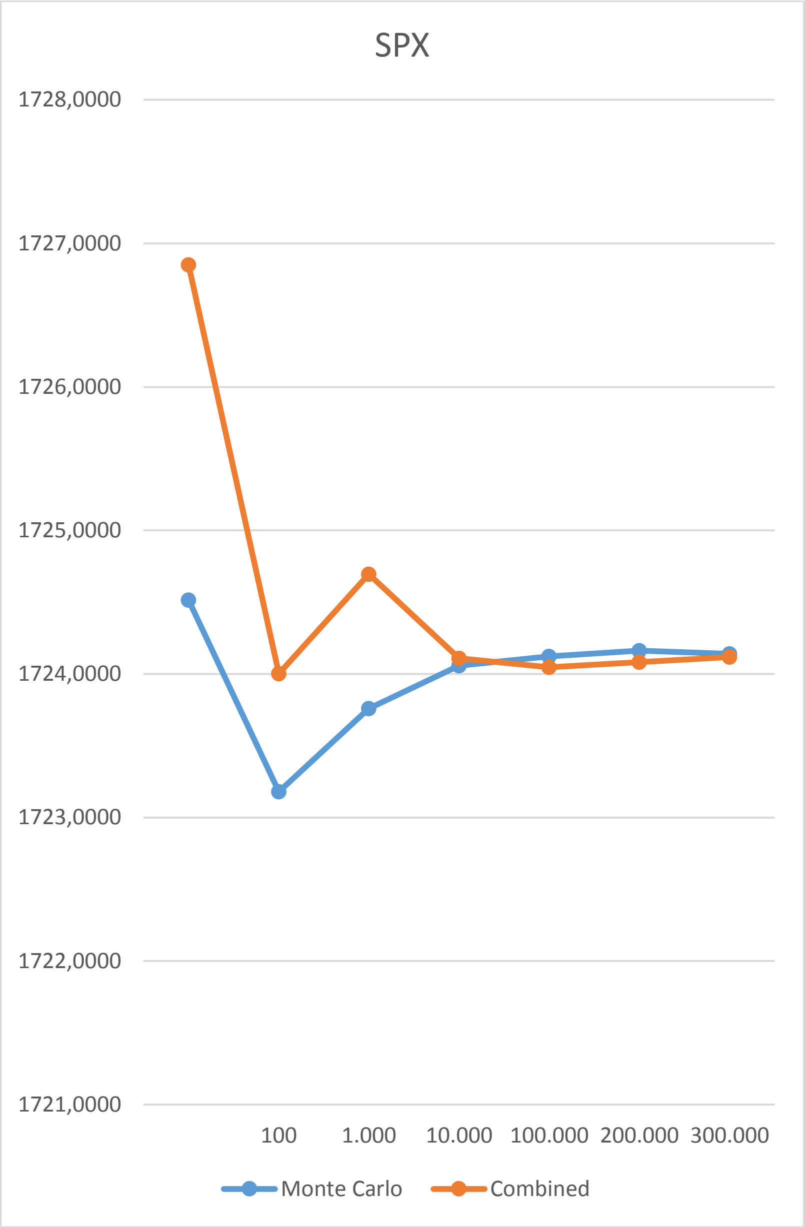
Table 4: Simulation Prices European Call in-the-money with Dividends (SPX)

S0 = 2223.43, K=500, T=0.0833 (3 weeks), r=1.56 %, σ = 4.67% ,I = # of iterations		
BSM Analytical	**General Monte Carlo**	**Combined**
1724.081	1724.513 (I = 100) 1724.758 (I = 10,000) 1724.059 (I = 100,000) 1724.122 (I = 200,000) 1724.141 (I = 400,000)	1726.850 (I = 100) 1724.694 (I = 10,000) 1724.109 (I = 100,000) 1724.046 (I = 200,000) 1724.118 (I = 400,000)

When SPX option is in-the-money, the number of iterations performed matters, see Table 4. In fact, when I=100 the biggest difference is the difference between the BSM Model (1724.081) and the Combined Model (1726.850) and its value is 0.16%. With an increase in number of iterations, the difference is smaller. For example, when I=400,000 – the absolute difference between the three methods is 0.004 %. So, it is possible to conclude that the increase in number of iterations can decrease the absolute difference of market value between the three models from 0.16 % to 0.0004 %.

Chart 2 is used to show the convergence of The General Monte Carlo Method and The Combined Method estimates again due to Law of Large Numbers.

Chart 2: In-the-Money SPX Option Contract



When the SPX Option Contract is at-the-money and the option will expire in three weeks, the difference in estimated market value of contract between the three models is 0.15%. Furthermore, if SPX option is going to expire in 36 weeks – the absolute difference between the option market price estimated by three models is 0.16%. So, the results from Table 5 suggest that the fair market value of the at-the-money SPX option can be efficiently estimated using all three models since they have similar performance.

Table 5: Comparative Option Values SPX

S0 = 2223.4, r=1.56%, σ = 4.67 %, Iterations = 400,000

	Time to Expiration (in weeks)											
Strike Price	3	9	18	36	3	9	18	36	3	9	18	36
	BSM				**Monte Carlo Simulation**				**Combined**			
100	2124.27	2123.82	2124.21	1231.20	2123.62	2123.92	2124.35	2125.19	2123.60	2123.87	2124.09	2125.11
250	1974.12	1974.40	1975.37	1977.30	1973.82	1974.51	1975.52	1977.51	1973.77	1974.34	1975.31	1977.12
500	1724.08	1725.38	1727.32	1731.17	1724.14	1725.48	1727.46	1731.38	1724.11	1725.41	1727.44	1977.12
2223.43	13.45	25.29	38.62	60.58	13.47	25.33	38.69	60.68	13.47	25.33	38.73	60.59
2600	0.00	0.00	0.01	0.04	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.04

Case 3: Empirical Work and Methodology for American Call Options with Dividends

In order to valuate American call options that pay dividends, I will use Apple Option Contract data. The General Monte Carlo Method and Least-Squares Monte Carlo Method are used and compared to mid-point of bid-ask spread. Due to the fact that American Options that pay dividends can be exercised anytime until their maturity, it is not possible to use The Black Scholes Model. The dividend discount model has to be used again in order to figure out the current underlying stock price.

By observing data from Table 6, it is possible to notice that when I =100 the Least-Squares Monte Carlo (48.868) is closer to the Mid-point (49.22) than The General Monte Carlo (46.909). As the number of iteration increase, the difference between the Least-Squares Monte Carlo and General Monte Carlo decreases significantly. This can be illustrated graphically by Chart 3.

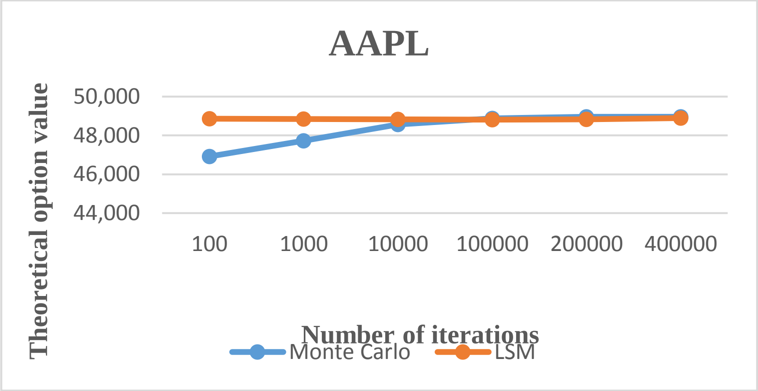
Table 6: Simulation Prices American Call in-the-money with Dividends

(AAPL)

1It is interesting to point out that it takes more iterations for stock options (GOOGLE and AAPL) to converge. Chart 1 and Chart 3 suggests that the minimum number of iterations should be at least 100,000. On the other hand, Chart 2 shows the optimal number of iterations for SPX Index Option Contract is approximately 10,000. So, it is possible to conclude that index options need less iterations to converge due to the fact that index represents the average value of 500 US companies and it is less volatile than the option on particular underlying stock such as Google or Apple.

Chart 3: In-the-Money AAPL Option Contract

S0 = 138.1979 , K=90, T=0.16, σ=62.50 %, r= 1.56 % , I = # of iterations		
Mid –point	**General Monte Carlo**	**LSM**
49.22 (given)	46.909 (I = 100) 48.560 (I = 10,000) 48.883 (I = 100,000) 48.955 (I = 200,000) 48.957 (I = 400,000)	48.868 (I = 100) 48.837 (I = 10,000) 48.815 (I = 100,000) 48.831 (I = 200,000) 48.894 (I = 400,000)



Through this empirical work, it is shown that The General Monte Carlo performs equally efficient as the other two models for a European call option that pays dividends and American call options that do not pay dividends. On the other hand, The General Monte Carlo performs the worst in comparison to the Least-Squares Monte Carlo Method in the valuation of American call options that pay dividends. Furthermore, it is shown that the number of iterations have impact on theoretical value of The Monte Carlo Method, The Combined Method and The Least-Squares Monte Carlo. In general, it is possible to notice that all those methods benefit with the increase in the number of iterations.

CONCLUSION

The main aim of this thesis was to investigate the use of the four different valuation models in pricing European and American call options. In fact, I have used Google Option Contract, SPX Contract and Apple Contract in order to conduct the valuation experiments. In fact, I observed three different cases and my results suggest that The Black Scholes Model, The General Monte Carlo Method and The Combined Model perform equally well for European Option Contract with dividends (SPX) and American Option Contract with no dividends (GOOGLE). On the other hand, I have found that The Least-Squares Monte Carlo is better estimator of theoretical value of American call option with dividends (APPLE) in comparison to The General Monte Carlo Method. Furthermore, it is important to emphasize that the number of iterations count plays an important role since it improves significantly option pricing methods. It is shown that valuation algorithms require less iterations for index option (SPX) than for the stock options (10,000 iterations vs. 100,000) due to the lower volatility level of SPX index. However, the results of pricing models are theoretical values and that they might differ from the real market values for particular option contracts. The disadvantage of these pricing model is the failure to incorporate the transaction costs in calculations. Rubinstein (1981) states that "although tests in area of options are not the most efficient, accurate, and conclusive, they have paved the way to better understanding not only of the behavior of option prices but also of stock prices" (p.2). I hope that my research will be a useful aid to individuals who would like to learn more about option pricing. The greatest limitation to this research was collecting data - option data access is restricted and it requires data purchase. An additional limitation is the fact that I have used The General Monte Carlo Method that assumes that the underlying stock follows the normal distribution. To further my research, I plan to extend the comparison by using The Geometric Monte Carlo Model since the major assumption of this model is that underlying stock follows a log-normal distribution. Also, I would like to explore the use of The Monte Carlo Method in valuation of Asian and other exotic option contracts.

UPOTREBA MONTE CARLO SIMULACIJE U VALUACIJI EVROPSKIH I AMERIČKIH CALL OPCIJA

MA Gorica Malešević

Apstrakt: Ova studija ispituje metode koje se koriste pri vrednovanju evropskih i američkih call opcija. Opcije su finansijski instrumenti koji igraju bitnu ulogu u finansijskoj industiji i koriste se za zaštitu, špekulaciju i arbitražu. S obzirom da se opcije koriste u investiranju, postoji velike potreba za preciznim metodama vrednovanja. U prošlosti su opcije smatrane nejasnim i „mračnim“ zbog nedostatka tehnika vrednovanja. Međutim, 1973 godine je otkriven prvi metod vrednovanja opcija Black-Scholes Model, te je trgovanje opcijama eskaliralo. U ovoj studiji fer tržišna vrijednost S&P 500 indeksa, Google opcijskog ugovora i Apple opcijskog ugovora se izračunava koristeći sledeće tehnike: Black-Scholes Model, Generalna Monte Carlo Simulacija, Kombinovani Monte Carlo Metod i Least-Squares Monte Carlo. Rezultati prethodno navedenih modela su upoređeni kako bi se odredio najbolji metod vrednovanja.

Ključne riječi: *Opcije, Vrednovanje, Monte Carlo Simulacija*

Appendix

Code 1.

```
#Combination of Monte Carlo Method and Black-Scholes-Merton Model
#Used to valuate only European call and American call (no div) options
# Code originally created by Yves Hilpisch
#Code can be found in book (Python for Finance) edition 2014

import numpy as np
#Parameteres
S0 = 2223.43 #Underlying Stock Price
K = 2600 #Strike Price
T = 0.25 #Time until experation
r =0.0156 #Risk-free interest rate
sigma =0.0467 #Volatility
I = 400000 #Iteration number

# Valuation Algorithm
z = np.random.standard_normal(I) # generating pseudorandom numbers
ST = S0 * np. exp((r - 0.5 * sigma ** 2) * T + sigma * np.sqrt(T) * z)
# index values at maturity
hT = np.maximum(ST - K, 0) # inner values at maturity
C0 = np.exp(-r * T) * np.sum(hT) / I # Monte Carlo estimator
print(C0)
```

Code 2.

```
#Monte Carlo Valuation of European Call and
#American call (no div) by using Numpy package(vectorization)
# Code originally created by Yves Hilpisch
#Code can be found in book (Python for Finance) edition 2014

import math
import numpy as np
from time import time
np.random.seed(20000)
t0 = time()


# Parameters used
S0 = 2223.43 #Underlying stock price
K = 2600 #Strike price
T = 1.0 #Time to experation
r =0.0156 #Risk-Free interest rate
sigma= 0.0467 #Volatility
M = 50;#Number of Time step intervals
dt = T / M; #time interval length
I=400000 #total number of iterations

# Simulating I paths with M time steps
S = np.zeros((M + 1, I))
S[0] = S0
for t in range(1, M + 1):
    z = np.random.standard_normal(I) #generating pseudo random number
    S[t] = S[t - 1] * np.exp((r - 0.5 * sigma ** 2) * dt
                             + sigma * math.sqrt(dt) * z)

# Calculating the Monte Carlo estimator
C0 = math.exp(-r * T) * np.sum(np.maximum(S[-1] - K, 0)) / I
print(C0)
```

Code 3.

```
#LSM Model
## Code originally created by Yves Hilpisch - Code can be found in book (Python for Finance) edition 2014
import numpy as np
import numpy.random as npr
def gbm_mcs_amer(K, option='call'):
    S0 = 837.17 #initial parameters
    K = 837.17
    T = 1.0
    r =0.0156
    sigma= 0.271504508
    M = 50
    dt = T/M
    I=400000
    df = np.exp(-r * dt)
     def gen_sn(M, I, anti_paths=True, mo_match=True):

        if anti_paths is True:
            sn = npr.standard_normal((M + 1, I / 2))
            sn = np.concatenate((sn, -sn), axis=1)
        else:
            sn = npr.standard_normal((M + 1, I))
        if mo_match is True:
            sn = (sn - sn.mean()) / sn.std()
        return(sn)
# simulation of index levels
    S = np.zeros((M + 1, I))
    S[0] = S0
    sn = gen_sn(M, I)
    for t in range(1, M + 1):
        S[t] = S[t - 1] * np.exp((r - 0.5 * sigma ** 2) * dt+ sigma * np.sqrt(dt) * sn[t])
# case-based calculation of payoff
    if option == 'call':
        h = np.maximum(S - K, 0)
    else:
        h = np.maximum(K - S, 0)
# LSM algorithm
    V = np.copy(h)
    for t in range(M - 1, 0, -1):
        reg = np.polyfit(S[t], V[t + 1] * df, 7)
        C = np.polyval(reg, S[t])
        V[t] = np.where(C > h[t], V[t + 1] * df, h[t])
# MCS estimator
    C0 = df * 1 / I * np.sum(V[1])
    return(C0)
```

Code 4.

```
#Monte Carlo Valuation of European Call and
#American call (no div) by using Numpy package(vectorization)
# Code originally created by Yves Hilpisch
#Code can be found in book (Python for Finance) edition 2014

import math
import numpy as np
from time import time
np.random.seed(20000)
t0 = time()


# Parameters used
S0 = 2223.43 #Underlying stock price
K = 2600 #Strike price
T = 1.0 #Time to experation
r =0.0156 #Risk-Free interest rate
sigma= 0.0467 #Volatility
M = 50;#Number of Time step intervals
dt = T / M; #time interval length
I=400000 #total number of iterations

# Simulating I paths with M time steps
S = np.zeros((M + 1, I))
S[0] = S0
for t in range(1, M + 1):
    z = np.random.standard_normal(I) #generating pseudo random number
    S[t] = S[t - 1] * np.exp((r - 0.5 * sigma ** 2) * dt
                              + sigma * math.sqrt(dt) * z)

# Calculating the Monte Carlo estimator
C0 = math.exp(-r * T) * np.sum(np.maximum(S[-1] - K, 0)) / I
print(C0)
```

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